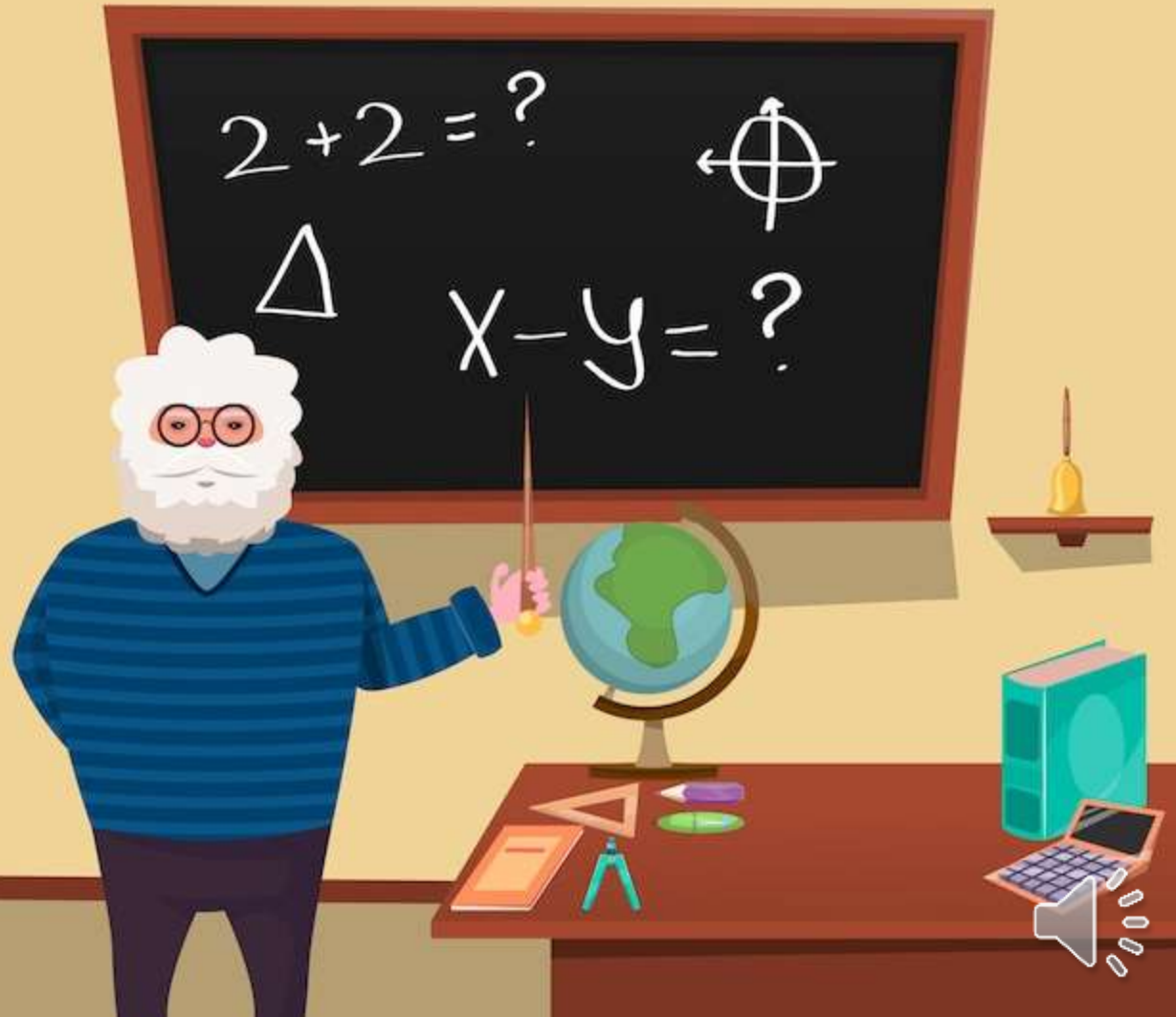
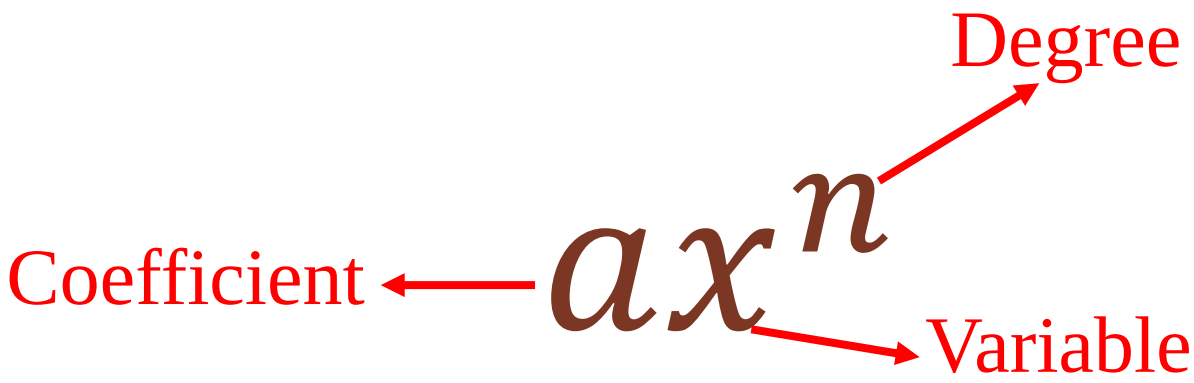
Algebraic Expressions

Part 1



Recall

❖ A monomial is in form of ax^n



Example: $-3x^2$; $2x$; $-\frac{1}{2}x^4$; ...

Remark: $x^0 = 1$

A monomial can have only the coefficient: -2 is a monomial of degree 0 since $-2 = -2x^0$

Recall

❖ The sum of two monomials is called binomial.

Example: $2x + 1$; $-3x^2 + 5$; $5x^6 - 3x^4$; $-2x - x^5$; ...

❖ The sum of more than two monomials is called polynomial.

Example: $x^3 - 2x + 1$; $-\frac{1}{2}x^{10} + 2x^5 - x^3$; ...

Recall

- The degree of a polynomial is the highest exponent of the variable after reducing the polynomial.

Example: $4x^3 - 2x^5 + x^2 + 1$ is a polynomial of degree 5

- Two monomials having same degree and same variable are called like monomials or similar monomials.

Example: x^3 and $-2x^3$; $5x^4$ and $\frac{1}{2}x^4$

Operations on polynomials

❖ Reducing and arranging a polynomial

Consider the polynomial:

$$P(x) = 2x^5 + 3 - 2x^5 + x^2 + 4x - 2x^2 - 3x + 6$$

Reducing a polynomial means simplifying the similar terms.

$$P(x) = (2x^5 - 2x^5) + (x^2 - 2x^2) + (4x - 3x) + (3 + 6)$$
$$= -x^2 + x + 9$$

This step can be done mentally

Operations on polynomials

❖ Addition of polynomials

Consider the polynomials:

$$P(x) = 2x^2 + 4x + 8 \text{ and } Q(x) = 3x^3 - 5x^2 + 2x - 5$$

Addition (or sum) of $P(x)$ and $Q(x)$:

$$P(x) + Q(x) = 2x^2 + 4x + 8 + 3x^3 - 5x^2 + 2x - 5$$

$$= 3x^3 + 2x^2 - 5x^2 + 4x + 2x + 8 - 5$$

$$= 3x^3 - 3x^2 + 6x + 3$$

$$P(x) + Q(x) = 3x^3 - 3x^2 + 6x + 3$$

Operations on polynomials

❖ Subtraction of polynomials

Consider the polynomials:

$$P(x) = 2x^2 + 4x + 8 \text{ and } Q(x) = 3x^3 - 5x^2 + 2x - 5$$

Subtraction (or difference) of $P(x)$ and $Q(x)$:

$$P(x) - Q(x) = 2x^2 + 4x + 8 - (3x^3 - 5x^2 + 2x - 5)$$

$$= 2x^2 + 4x + 8 - 3x^3 + 5x^2 - 2x + 5$$

$$= -3x^3 + 2x^2 + 5x^2 + 4x - 2x + 8 + 5$$

$$= -3x^3 + 7x^2 + 2x + 13$$

$$P(x) - Q(x) = -3x^3 + 7x^2 + 2x + 13$$

Operations on polynomials

❖ Multiplication of polynomials

Consider the polynomials:

$$P(x) = 2x^2 + 4x + 8 \text{ and } Q(x) = 3x^3 - 5x^2 + 2x - 5$$

Multiplication (or product) of $P(x)$ and $Q(x)$:

$$P(x) \times Q(x) = (2x^2 + 4x + 8)(3x^3 - 5x^2 + 2x - 5)$$

$$= 6x^5$$

Operations on polynomials

❖ Multiplication of polynomials

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Multiplication (or product) of $P(x)$ and $Q(x)$:

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$$= 6x^5 - 10x^4$$

Operations on polynomials

❖ Multiplication of polynomials

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Multiplication (or product) of $P(x)$ and $Q(x)$:

$$P(x) \times Q(x) = (2x^2 + 4x + 8)(3x^3 - 5x^2 + 2x - 5)$$

$$= 6x^5 - 10x^4 + 4x^3$$

Operations on polynomials

❖ Multiplication of polynomials

Consider the polynomials:

$$P(x) = 2x^2 + 4x + 8 \text{ and } Q(x) = 3x^3 - 5x^2 + 2x - 5$$

Multiplication (or product) of $P(x)$ and $Q(x)$:

$$P(x) \times Q(x) = (2x^2 + 4x + 8)(3x^3 - 5x^2 + 2x - 5)$$

$$= 6x^5 - 10x^4 + 4x^3 - 10x^2$$

Operations on polynomials

❖ Multiplication of polynomials

Consider the polynomials:

$$P(x) = 2x^2 + 4x + 8 \text{ and } Q(x) = 3x^3 - 5x^2 + 2x - 5$$

Multiplication (or product) of $P(x)$ and $Q(x)$:

$$P(x) \times Q(x) = (2x^2 + 4x + 8)(3x^3 - 5x^2 + 2x - 5)$$

$$= 6x^5 - 10x^4 + 4x^3 - 10x^2 + 12x^4$$

Operations on polynomials

❖ Multiplication of polynomials

Consider the polynomials:

$$P(x) = 2x^2 + 4x + 8 \text{ and } Q(x) = 3x^3 - 5x^2 + 2x - 5$$

Multiplication (or product) of $P(x)$ and $Q(x)$:

$$P(x) \times Q(x) = (2x^2 + 4x + 8)(3x^3 - 5x^2 + 2x - 5)$$

$$= 6x^5 - 10x^4 + 4x^3 - 10x^2 + 12x^4 - 20x^3$$

Operations on polynomials

❖ Multiplication of polynomials

Consider the polynomials:

$$P(x) = 2x^2 + 4x + 8 \text{ and } Q(x) = 3x^3 - 5x^2 + 2x - 5$$

Multiplication (or product) of $P(x)$ and $Q(x)$:

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$$= 6x^5 - 10x^4 + 4x^3 - 10x^2 + 12x^4 - 20x^3 + 8x^2$$

Operations on polynomials

❖ Multiplication of polynomials

Consider the polynomials:

$$P(x) = 2x^2 + 4x + 8 \text{ and } Q(x) = 3x^3 - 5x^2 + 2x - 5$$

Multiplication (or product) of $P(x)$ and $Q(x)$:

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$$= 6x^5 - 10x^4 + 4x^3 - 10x^2 + 12x^4 - 20x^3 + 8x^2 - 20x$$

Operations on polynomials

❖ Multiplication of polynomials

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Multiplication (or product) of $P(x)$ and $Q(x)$:

$$P(x) \times Q(x) = (2x^2 + 4x + 8)(3x^3 - 5x^2 + 2x - 5)$$

$$= 6x^5 - 10x^4 + 4x^3 - 10x^2 + 12x^4 - 20x^3 + 8x^2 - 20x + 24x^3$$

Operations on polynomials

❖ Multiplication of polynomials

Consider the polynomials:

$$P(x) = 2x^2 + 4x + 8 \text{ and } Q(x) = 3x^3 - 5x^2 + 2x - 5$$

Multiplication (or product) of $P(x)$ and $Q(x)$:

$$P(x) \times Q(x) = (2x^2 + 4x + 8)(3x^3 - 5x^2 + 2x - 5)$$

$$= 6x^5 - 10x^4 + 4x^3 - 10x^2 + 12x^4 - 20x^3 + 8x^2 - 20x$$
$$+ 24x^3 - 40x^2$$

Operations on polynomials

❖ Multiplication of polynomials

Consider the polynomials:

$$P(x) = 2x^2 + 4x + 8 \text{ and } Q(x) = 3x^3 - 5x^2 + 2x - 5$$

Multiplication (or product) of $P(x)$ and $Q(x)$:

$$P(x) \times Q(x) = (2x^2 + 4x + 8)(3x^3 - 5x^2 + 2x - 5)$$

$$= 6x^5 - 10x^4 + 4x^3 - 10x^2 + 12x^4 - 20x^3 + 8x^2 - 20x$$
$$+ 24x^3 - 40x^2 + 16x$$

Operations on polynomials

❖ Multiplication of polynomials

Consider the polynomials:

$$P(x) = 2x^2 + 4x + 8 \text{ and } Q(x) = 3x^3 - 5x^2 + 2x - 5$$

Multiplication (or product) of $P(x)$ and $Q(x)$:

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$$= 6x^5 - 10x^4 + 4x^3 - 10x^2 + 12x^4 - 20x^3 + 8x^2 - 20x$$
$$+ 24x^3 - 40x^2 + 16x - 40$$

Operations on polynomials

❖ Multiplication of polynomials

Consider the polynomials:

$$P(x) = 2x^2 + 4x + 8 \text{ and } Q(x) = 3x^3 - 5x^2 + 2x - 5$$

Multiplication (or product) of $P(x)$ and $Q(x)$:

$$P(x) \times Q(x) = (2x^2 + 4x + 8)(3x^3 - 5x^2 + 2x - 5)$$

$$= 6x^5 - 10x^4 + 4x^3 - 10x^2 + 12x^4 - 20x^3 + 8x^2 - 20x$$
$$+ 24x^3 - 40x^2 + 16x - 40$$
$$= 6x^5 + 2x^4 + 8x^3 - 42x^2 - 4x - 40$$

Operations on polynomials

❖ Multiplication of polynomials

Consider the polynomials:

$$P(x) = 2x^2 + 4x + 8 \text{ and } Q(x) = 3x^3 - 5x^2 + 2x - 5$$

Multiplication (or product) of $P(x)$ and $Q(x)$:

$$P(x) \times Q(x) = (2x^2 + 4x + 8)(3x^3 - 5x^2 + 2x - 5)$$

$$P(x) \times Q(x) = 6x^5 + 2x^4 + 8x^3 - 42x^2 - 4x - 40$$

Operations on polynomials

Remark:
The degree of the sum (or the difference) of two polynomials is less than or equal the highest degree of the two polynomials.

Example:
$P(x) = x^2 + 2x + 1\ (degree = 2)$
$Q(x) = x^3 - 3x (degree = 3)$
$R(x) = -x^2 + 5\ (degree = 2)$
$P(x) + Q(x) = x^3 + x^2 - x + 1$ is of degree 3 (same as *Q(x)*)
$P(x) + R(x) = 2x + 6$ is of degree 1 (less than that of both *P(x)* and *R(x)*)

Operations on polynomials

Remark:
The degree of the product of two polynomials is equal to the sum of the degrees of the two polynomials.

Example:

$P(x) = x^2 + 2x + 1\ (degree = 2)$

$Q(x) = x^3 - 3x (degree = 3)$

$R(x) = -x^2 + 5\ (degree = 2)$

$P(x) \times Q(x) = x^5 + 2x^4 - 2x^3 - 6x^2 - 3$ is of degree 5 (2+3)

$P(x) \times R(x) = -x^4 - 2x^3 + 4x^2 + 10x + 5$ is of degree 4 (2+2)

Consider the two polynomials:

$P(x) = 2x^4 - 3x^2 + 4x - 5$

$Q(x) = -2x^4 + 5x^2 - 3$

Calculate $P(x) + Q(x)$; $P(x) - Q(x)$ and $P(x) \times Q(x)$.

$P(x) + Q(x) = 2x^2 + 4x - 8$

$P(x) - Q(x) = 4x^4 - 8x^2 + 4x - 2$

$P(x) \times Q(x) = -4x^8 + 16x^6 - 8x^5$
$- 11x^4 + 20x^3 - 16x^2 - 12x + 15$